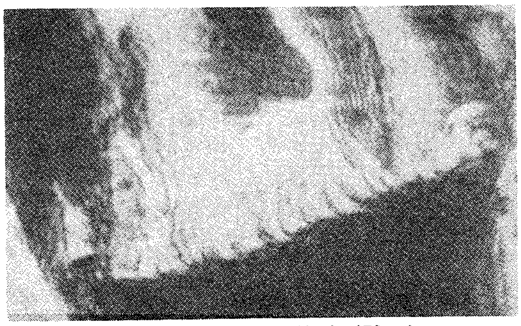

Fig. 2 AP (polycrystalline), 450 psi.

back lighting of the opaque specimen. Surface definition was poor in the interface region between the dark schlieren and the solid-phase surface. Observable from the motion pictures was an apparent motion of the solid-phase surface just below the burning surface. This was attributed to the reflections from the downstream bomb window. In the SC-UHP-AP specimen, this motion was not observed, since the transparent crystal allowed laser light transmission, which overpowered any such reflections.

Smoke absorption appeared to be a major problem, particularly with the limited-power laser light source. Light absorption by the smoke resulted in a darkened area, which was superimposed on any light-dark shift resulting from density gradients and confused the record interpretation. On the right-hand edge of Fig. 2, the smoke was evidenced by the gray region in the gas phase. Smoke accumulation was minimized by controlled purge rate.

Additional schlieren were taken at 1000 and 2000 psi. Details and photographs are presented in Ref. 4. There were two major differences between the 1000- and 500-psi results. The first was a near-constant periodic pulsing of the burning process. At regular intervals, a very thin layer of smoke would move upward from the immediate vicinity of the surface all along the width. This smoke layer was everywhere parallel to the surface locally; that is, it reflected the instantaneous surface contour. This result was similar to those reported by Murphy and Netzer.[6] They reported a "thermal pulsing" when their aperture was positioned to detect vertical density gradients. Boggs and Zurn[7] similarily reported an accumulation and shedding of unreacted products on the surface of potassium-doped AP crystals leading to a "stop-and-go" burning characteristic. The second difference was the lack of light-dark schlieren shifts across and just above the surface. This indicated little or no density gradients (i.e., approximately constant temperature) across the surface with uniform burning. Murphy and Netzer[6] reported similar results with "almost uniform color in the gases just above the surface." These results imply that surface reaction sites are very small or nonexistent at 1000 psi.

At 2200 psi, the surface locally appeared to be nonuniform, with large-scale turbulence very close to the surface. Density gradients were observed which extended to the burning surface. However, the smoke and fringing near the surface prevented a consistent determination of the size of the surface reaction sites. The pulsing behavior observed at 1000 psi was not observed at 2200 psi.

General conclusions concerning the use of laser schlieren for solid-propellant studies are as follows: 1) The basic feasibility of using an optically active aperture laser schlieren system for high-pressure solid-propellant combustion study was demonstrated. 2) The major advantage of this schlieren system over conventional schlieren systems is the elimination of self-luminous interference. 3) For applications where the self-luminous problem is nonexistent (i.e., shock pattern studies, etc.), a cw laser schlieren in general would be inferior to conventional schlieren systems. Conventional color schlieren has the added advantage over this system in that schlieren effects can be distinguished more readily from variable light absorption by smoke. 4) System resolution was limited to approximately 60 μ, primarily by fringing. The fringing could, perhaps, be minimized or eliminated by using a bonded prism set and mirrors in place of the lenses. Another method of improving the schlieren quality would be to increase system sensitivity by using a longer focal length schlieren and/or a greater quartz crystal angle. This would allow the use of a polaroid axis, which would provide a darker background, thereby "darkening out" the fringes so that they would not be seen on the film.

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Comparison of Reynolds Stress Diagnostics by Fixed and Rotating Probes

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Introduction

THREE-DIMENSIONAL turbulent-boundary-layer (3DTBL) flows continue to generate strong interest in fluid-mechanics research. The structure and details of such flowfields are of interest, and Reynolds stress measurements can be very useful in verification of empirical mixing length or eddy viscosity models and of computational schemes. Although only two Reynolds stresses, viz., the streamwise and transverse shear stresses, generally are included in a 3DTBL analysis, details on the nature and development of all six of the unknown Reynolds stresses have considerable value. Some more recent computational schemes attempt to include one or more of the Reynolds stress terms usually neglected in a boundary-layer-type analysis, and hence information on all of the

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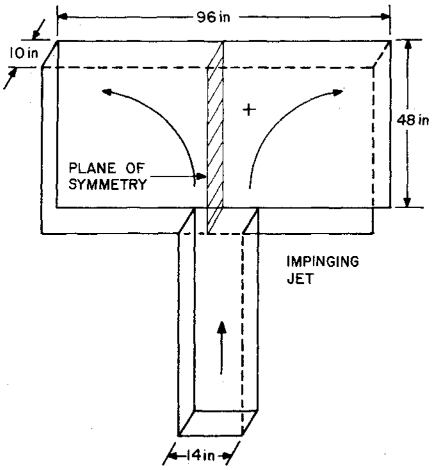

Fig. 1 Schematic of test apparatus.

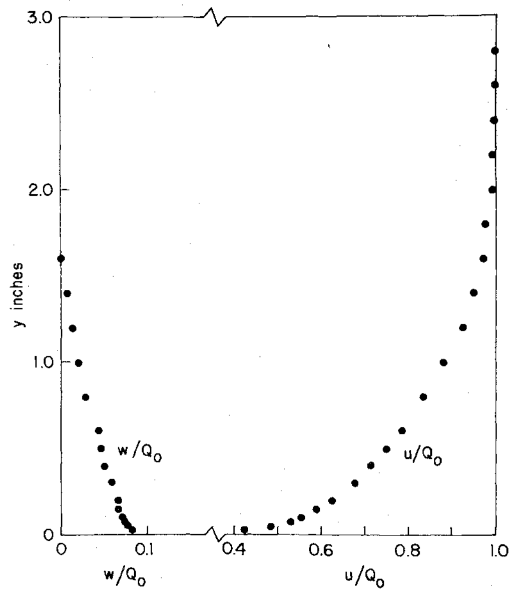

Fig. 2 Velocity profile at test station.

Reynolds stresses might be necessary in evaluating such schemes for predicting internal channel flows or turbomachinery passage flows.

Although some Reynolds stresses are relatively simple to measure, the measurement of all six such stresses is a significantly difficult undertaking requiring substantial equipment, development of technique, and in some cases substantial data reduction. In the course of investigating some 3DTBL flows, the authors have had a good opportunity to evaluate thoroughly the two generally accepted methods of Reynolds stress measurement by hot wires/films. The substantial effort involved in obtaining such experience is illustrated by the following experiment and appraisal.

Experiment

All six unknown elements of the Reynolds stress tensor were measured through a pressure-driven 3DTBL with moderate skewing or transverse flow relative to the freestream direction by two separate and independent hot-wire/film methods.

The first method consists of rotating two single wire or film probes and tracing signals with respect to angular position. Fujita and Kovasznay[1] first developed this method to measure three of the Reynolds stresses ($\overline{u^2}$, $\overline{w^2}$, and $\overline{uw}$) with a sensor element rotating in a plane parallel to the surface bounding the flow. Subsequently, Bissonnette and Mellor[2] developed an analysis that showed how, by rotating a sensor at an angle to the physical plane bounding the flow and about its axis, the remaining three Reynolds stresses($\overline{v^2}$, $\overline{uv}$, and $\overline{vw}$ could be evaluated in a boundary-layer flow. In this rotating-probe method, one traces the sensor output with angular position (for example, on an x-y recorder), and analysis of the sensor response equations[1,2] yields a set of simultaneous linear equations for the stress terms. Because of the difficulty in data acquisition and the subjectivity of interpretation of such data, a regression analysis is necessary in order to use a larger spectrum of the recorded data to reduce the effects of reading uncertainties as well as experimental uncertainties. Typically, 12 to 20 data points were used in the regression analysis. Actually, the slant wire analysis[2] would allow a system of simultaneous linear equations for the prediction of all six of the Reynolds stress terms. In practice, more consistent data are obtained by using the straight-wire data to determine three of these stresses, with subsequent use of these three known stresses as input to the slant-wire analysis to determine the unknown remaining three Reynolds stresses.

The second method used to measure these same Reynolds stresses was the method developed by Gessner[3] after the suggestion of Rodet.[4] In this method, four traverses of fixed x arrays are made through the boundary-layer orienting the probe with respect to the local boundary-layer velocity vector. Single traverses are made with a horizontal x array, a vertical x array, and two traverses are made with a slant x array (an array contained in a plane along the local flow direction and at 45° with the physical floor). In this method, the sensor rms signals are fed directly to a standard correlator/sum-difference unit, and the Reynolds stresses follow by simple calculations. A typical analysis of such data is given in Refs. 3 and 5.

In the method of Bissonnette and Mellor, a sensor calibrated in a two-dimensional flow is used to give the direction and magnitude of the local boundary-layer velocity vector at the same time as the rms signals are being generated with respect to angular position for the subsequent determination of the Reynolds stresses. In the method of Gessner, the velocity field must be determined independent of the Reynolds stress measurements, and this usually is done with the lower wire of the horizontal x array without undue difficulty.

Results

Figure 1 shows a schematic of the flow geometry, and Fig. 2 shows the velocity profile for the conditions studied. This was a typical low-speed incompressible pressure-driven 3 DTBL over a flat surface where the transverse flow was generated by the curvature of the freestream streamlines. Figure 3 shows the normal turbulent intensity parameters (the nondimensionalized Reynolds normal stresses), and Fig. 4 shows the shear turbulent intensity parameters (the nondimensionalized Reynolds shear stresses) measured by the two methods for the same nominal flow conditions at two different points in time.

In consideration of the difficulty of such measurements, the agreement between the two sets of results is judged acceptable, with nominal lines drawn through the data showing good to acceptable agreement. The fixed-sensor, x-array method shows a more regular behavior of the results, possibly because rms averages are taken over a longer period of time for any sensor orientation, whereas in the rotating-sensor method the sweep rate, though small, allows a relatively short time for any discrete sensor orientation interval. Differences in the results of these two methods are representative of the kind of differences that were encountered in repeated measurements under the same nominal flow conditions by a given method.

The rotating-sensor method shows more scatter in the results, and there is a tendency to produce an occasional

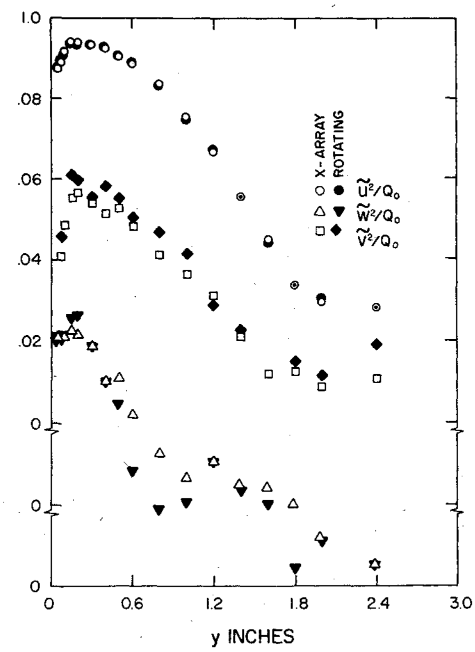

Fig. 3 Nondimensionalized normal Reynolds stresses.

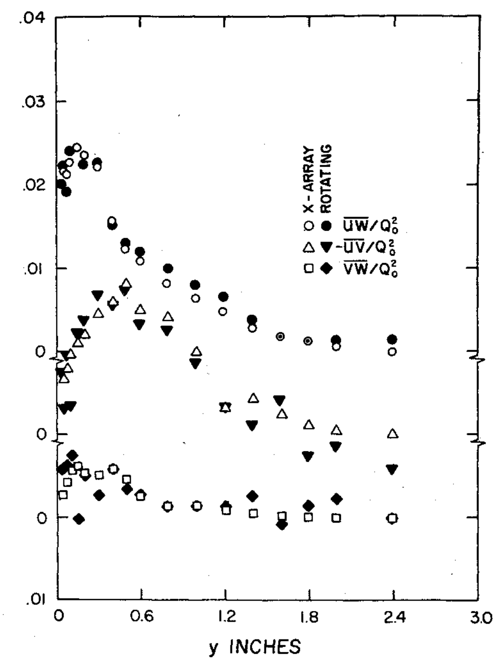

Fig. 4 Nondimensionalized shear Reynolds stresses.

specious stress value, particularly in the slant-sensor output. These typical results strongly suggest the need to obtain a profile of any given stress rather than rely on any single discrete data point. In this rotating-wire method, the horizontal wire gives the $\overline{u^2}$, $\overline{w^2}$, and $\overline{uw}$ stresses. These are seen to have better repeatability than the $\overline{v^2}$, $\overline{uv}$, and $\overline{vw}$ stresses that follow from the slant-wire output, with the earlier three stresses that follow from the slant-wire output, with the earlier three stresses also used as input. The uncertainties in these input stresses necessarily are reflected in the uncertainties of the $\overline{v^2}$, $\overline{uv}$, and $\overline{vw}$ stresses. Various exercises were attempted where all six Reynolds stresses were found using only the rotating-slant sensor output and a set of six linear simultaneous equations. In nearly all cases, there was substantially more scatter and much less confidence in this approach.

Conclusions

A comparison of all six terms of the Reynolds stress tensor measured in a pressure-driven 3DTBL by two independent methods is presented. In the course of two experiments, complete Reynolds stress tensor profiles have been measured independently for eight stations by the x-array method and 14 stations by the rotating-sensor method. Based on this extensive experience, a comparison of overall effort and some generalized comments on typical output can be made.

Agreement between the two methods is judged good. It becomes a matter of personal choice as to which method is preferred. Both methods require a considerable amount of time and development of laboratory technique. The rotating-sensor method, once the regression analysis and computer programing are developed, still requires substantial time in manually reading data sets for input to the regression analysis. The scheme cannot be automated entirely in that some discretion enters into determining local flow direction with angular position at maximum signal, etc. The x-array technique is time-consuming in that four traverses are required through the boundary layer but, on the other hand, requires very little data reduction, with the stresses coming almost directly from the correlator/sum-difference unit. Both methods measure stresses relative to local flow direction, a variable through the traverse, but a simple transformation will give the stress field relative to the freestream or body coordinate system.

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Symmetric Stiffness Matrix for Incompressible Hyperelastic Materials

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SOME materials have the characteristics that the stress tensor is derivable from a strain energy function and that the deformation occurs without an appreciable change in volume. These are called incompressible hyperelastic materials, which include the significant difficulty that in plane strain or axisymmetric problems the stress tensor is not determined by only the deformation. A hydrostatic pressure that does not affect the strain tensor must be considered to

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